Class 12 Mathematics – Chapter: Continuity and Differentiability

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1. Introduction

- Continuity and differentiability are foundational concepts in calculus.
- They describe smoothness and rate of change of functions.

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2. Continuity

- A function f(x)f(x)f(x) is continuous at x=ax = ax=a if:
  1. f(a)f(a)f(a) is defined.
  2. lim⁡ x→ af(x)\lim_{x\to a} f(x)limx→af(x) exists.
  3. lim⁡ x→ af(x)=f(a)\lim_{x\to a} f(x) = f(a)limx→af(x)=f(a).
- **Types of Discontinuity:**
  - Removable discontinuity
  - Jump discontinuity

- Infinite discontinuity

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3. Differentiability

- A function is differentiable at x=ax = ax=a if the derivative f'(a)f'(a)f'(a) exists.
- Derivative defined as:

  f'(a)=lim⁡h→0f(a+h)−f(a)hf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}f'(a)=h→0limhf(a+h)−f(a)
- Differentiability implies continuity, but not vice versa.

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4. Geometric Interpretation

- Continuity means the graph of f(x)f(x)f(x) has no breaks at x=ax = ax=a.
- Differentiability means the graph has a defined tangent at x=ax = ax=a.

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5. Derivative as a Function

- f'(x)f'(x)f'(x) gives the rate of change of fff at any xxx.
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If f'(x)f'(x)f'(x) exists for all xxx in an interval, fff is differentiable on that interval.

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6. Examples

- Polynomials are continuous and differentiable everywhere.
- Absolute value function is continuous everywhere but not differentiable at 0.

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7. Applications

- Used in optimization problems.
- Understanding behavior of functions.
- Physics: velocity as derivative of position.

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8. Exam Tips

- Know the definitions clearly.
- Practice proving continuity and differentiability.
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Solve problems on limits related to continuity.

- Understand examples of functions with/without differentiability.